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Structural Synthesis of "Parallel" Robots Generating Spatial Translation

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Abstract - This paper presents a tool, based on the mathematical Group Theory, for the synthesis of new parallel structure robots. By the kinematic principle of displacement subgroups intersection, a family of 3 degrees of freedom robots for pure spatial translation movements is conceived. One of the many possible implementations is also given as an example.

I. INTRODUCTION

The use of mathematical Group Theory, and more precisely of Lie's Groups, begins to become classical for modeling displacements of a rigid body ([1], [2], [3], [4], [5], [6], [7]).

If we call {D} the set of all possible displacements, it is proved, according to this Theory, that {D} has a group structure. ([1]). The most remarkable movements of a rigid body are then represented by subgroups of {D}. In the following text the coupling between two bodies will be referred to as "liaison", as it is an extension of the kinematic pair concept ([4]).

II. SERIALLY STRUCTURED ROBOTS GENERATING SPATIAL TRANSLATION

Manipulator robots generating spatial translation are frequently employed.

Translation movements are represented by the subgroup {T} of group {D}. For producing such movements, we usually put in series kinematic pairs (or mechanical liaisons) represented by non-empty subsets of the subgroup {T}. In practice, these kinematics pairs are either prismatic pairs (sliding pairs) or circular translation liaisons. These last liaisons can be typically illustrated by the two opposite bars of a plane hinged-parallelogram.

Prismatic pairs are represented by subgroups {T(D)} of rectilinear translations along any direction parallel to line D. Circular translation liaisons constitute a non-empty subset (or complex) of dimension 1 (degree of freedom 1) of the subgroup {T(P)} of plane translations parallel to plane P.

The putting in series physically of kinematic pairs is mathematically expressed by the composition product of the operators representing the component liaisons. We know from the Group Theory that for a given subgroup, the product of two elements pertains to the same subgroup. Hence, adding in series 3 mechanical liaisons represented by 1-dimensional complexes of the subgroup {T} usually generates a 3-dimensional liaison represented by a set of operators included in {T}.

The identity element for all the subgroups is the identity operator. In a given group, two equally dimensioned neighbourings of the identity operator of a subgroup are said to be equal. This, however is not perfectly correct from a mathematical point of view. The difference lies only in the limits of variation of the parameters. This is the reason why we claim to generate the spatial translation subgroup {T} by adding in series three prismatic pairs (for no coplanar directions).

The well known cartesian robot offers an example of these mathematical properties.

III. PARALLEL STRUCTURE ROBOTS

The main technical drawback of serial structure robots is due to the fact that each segment of the structure has to bear not only the weight of the manipulated object but also the load of the following segments generally carrying electric heavy and bulky servomotors. In addition, a serial structure is intrinsically inadequate for obtaining an exact positioning in space.

Parallel robots are better manipulators for the simple reason that positioning errors of the component liaisons are not additive and that motors can be placed solely at the back of the structure out of the working volume. In addition, rapid working rhythms are made possible by the low inertia of these kinds of robots.

Historically, parallel structure robots arose in 1949 when Gough [8] invented a machine for testing tires. Afterwards, Stewart [8] suggested the using of this mechanical structure as a flight simulator. Later, in the 1970s other applications were proposed. In recent years many researchers have addressed the subject, and parallel manipulating prototypes with 6 degrees of freedom have been developed. Hybrid manipulators (serial and parallel) have also been studied (see Merlet [9] for a complete bibliography).

The aim of this paper is to present a new geometrical model for a family of robots that has only 3 degrees of freedom for translational movements in space. These particular manipulators differ from those mentioned above by the fact that no rotational movements are considered.

The first device of this kind was conceived by R. Clavel ([10],[11]), who had the idea of chaining in parallel several cooperating mechanisms in order to actuate this given kind of movement. Our work develops this idea with the support of the Group Theory and carries out other applications.

IV. BASIC PRINCIPLES OF PARALLEL STRUCTURE ROBOTS ACCORDING TO THE GROUP THEORY

If two bodies are joined by two mechanisms, we can say that there are two parallel liaisons between the bodies. The relative movements are governed at the same time by the two liaisons, that is to say by the intersection liaison, in the mathematical sense of intersection set of two sets of displacement operators.

A. *Intersection of two subgroups*

An important theorem of the Group Theory tells that the intersection of two subgroups is still a subgroup. In order to generate spatial translation with parallel mechanisms, we are led to look for displacement subgroups, the intersection of which is the spatial translation subgroup. We will consider only the remarkable cases for which the intersection subgroup is strictly included in the two "parallel" subgroups. Actually there is only one case of this sort. It consists of two subgroups of the kind {X(w)} , which has dimension 4.

B. *The subgroup {X(w)}*

The tranformations of the subgroup {X(w)} are the operations which act on point M becoming M' according to the formula :

$$M \rightarrow M' = N + a\,u + b\,v + c\,w + \exp(h\, w \wedge)\, NM$$

(N, u, v, w) being a given orthogonal frame of reference and : a, b, c, h the 4 variable parameters of the subgroup. The corresponding movements constitute a set of rotations around an axis that translates spatially along the constant direction determined by w.

We recall that we have :

$$\exp(h\, w \wedge)\, NM = NM + \frac{h}{1!}\, w \wedge NM + \frac{h^2}{2!}\, w \wedge (w \wedge NM) + \frac{h^n}{n!}\, (w \wedge)^n\, NM + \ldots$$

$$= NM + \sin h\,(w \wedge NM) + (1 - \cos h)\, w \wedge (w \wedge NM)$$

This formula represents the rotation, of an angle h, of vector NM around the axis (N, w).

C. *Mechanical Generators of {X(w)}*

Physical implementations of {X(w)} mechanical liaisons can be obtained by putting in series kinematic pairs represented by subgroups of {X(w)}.

In this first approach, we will consider only 1-degree of freedom kinematic pairs. They are :

- the revolute pair , which we shall denote with the symbol R
- the prismatic pair P
- the screw pair H

Any series of 4 of these pairs satisfying the following geometrical conditions makes up a mechanical generator of the subgroup {X(w)}:

1) the rotation axis and the screw axis are parallel to the given vector w
2) there is no passive mobility in the series.

The subgroups of the {X(w)} group are :

{T(D)}	rectilinear translation ∀ line D
{R(C,u)}	rotation around an axis ∀ point C, u // w
{H(C,u,p)}	helicoidal movement ∀ point C , u // w , ∀ pitch p
{T(P)}	planar translation ∀ plan P
{C(B,u)}	cylindric movement ∀ point B , u // w
{T}	spatial translation
{G(P)}	planar sliding on plane P ⊥ w
{Y(u,p)}	product of a plane translation ⊥ u and an helicoidal movement around an axis // u , ∀ pitch p.

It would take too long to give all the possible cases of passive mobility for mechanisms, but some typical examples are :

- two screws have the same axis and the same pitch
- three prismatic pairs are parallel to the same plane
- four revolute pairs have parallel axes .

We will also not consider the trivial generation of spatial translation by 3 prismatic pairs not lying in the same plane. In this case the additional R or H pairs do not move and are therefore useless.

The complete list of combinations of 1-degree of freedom kinematic pairs generating the subgroup {X(w)} and satisfying the conditions given above is :

with two Ps :	RRPP	HRPP	RHPP	HHPP
	RPRP	RPHP	HPRP	HPHP
	PRRP	PRHP	PHHP	
	RPPR	RPPH	HPPH	
with one P :	PRRR	PRRH	PRHR	PHRR
	PRHH	PHRH	PHHR	PHHH
	RRPR	HRPR	RHPR	RRPH
	HHPR	HRPH	RHPH	HHPH
with no P :	RRRH	RRHR	RRHH	RHRH
	RHHR	HRRH	HHHR	HHRH
	HHHH			

The inverse combinations must be considered as well.

The cylindric pair C combines in a compact way a prismatic pair and a revolute pair : C = PR , RP , PH , HP , RH , HR,HH .

It is useful to consider mechanisms starting with a cylindric pair. We shall then list the combinations ! :

CPP CRP CHP CPR CPH CRR CRH CHR CHH

It is also interesting to consider the circular translation liaisons generated in a plane-hinged parallelogram. Their corresponding symbol is Pa. The equivalent combinations are then obtained by replacing P with Pa.

V. CLAVEL'S DELTA ROBOT INTERPRETATION

For producing spatial translation it suffices to place in parallel two mechanical generators of the subgroups {X(w)} and {X(w')} , w ≠w', between a mobile platform and a fixed platform.

If we wish to have a robot only with fixed motors, we need three generators of the three subgroups {X(w)} , {X(w')} , {X(w'')} , w ≠ w', w' ≠ w'', w'' ≠ w .

{X(w)} ∩ {X(w')} = {T}
{X(w')} ∩ {X(w'')} = {T}
{X(w'')} ∩ {X(w)} = {T}

This is the solution adopted by Clavel for his Delta robot (fig. 1), which implements three

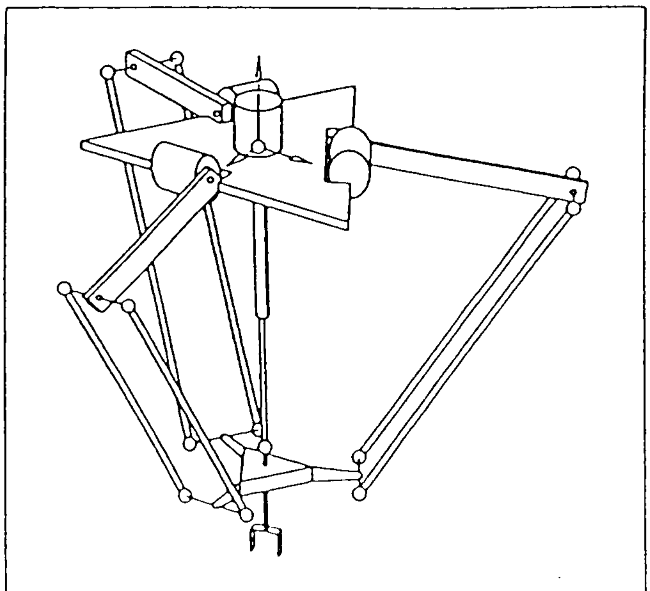

Fig. 1. Geometry of the DELTA4 robot

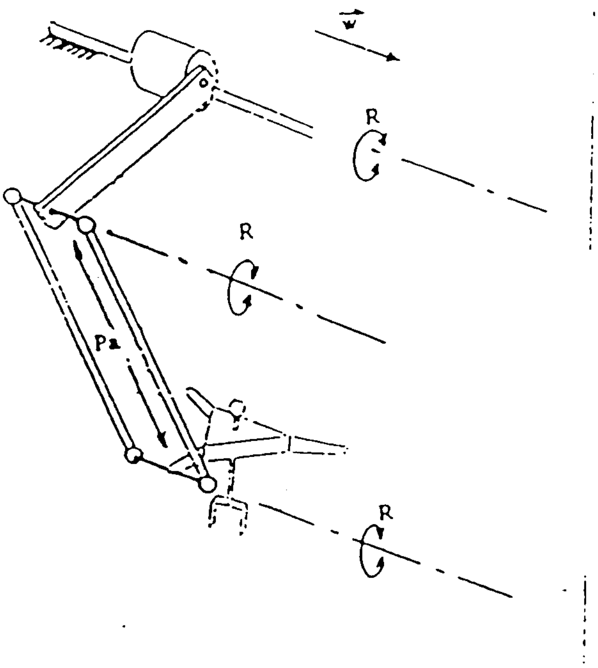

Fig. 2.

generators of the type RRPaR (fig.2) disposed in a symmetrical way (rotational symmetry of a third of a distance round, around an axis).

The result of our analysis is that Clavel's robot belongs to a wide family based on the same kinematic principles.

VI. EXAMPLE OF A NEW ROBOT

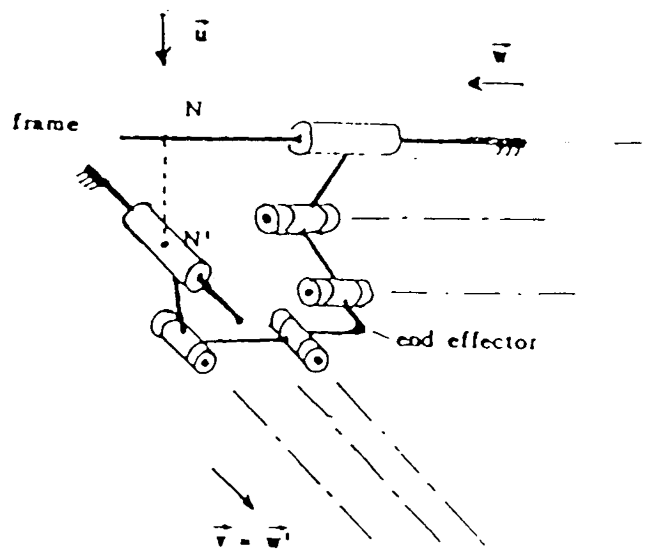

Fig 3. New parallel structure robot

The cylindric pair C can be motorized in a quasi-fixed way. Therefore, we plan a new robot with two cooperating arms of the CRR kind. We choose w'= v ⊥ w (fig. 3).

Let NN' be the common perpendicular to the fixed axis of the C pairs : NN' = p u . In order to have simpler calculations, we suppose the existence of a simultaneous streched configuration of the two arms (in a vertical position for example). We will use this as an initial configuration (fig. 4) for carrying out the geometrical model of the robot. In this configuration, the cylindric pair and the revolute pair axes of the first arm CRR intersect the common perpendicular at points N, A, B : NA = r u , AB = s u .For the second arm CRR : N'A' = r' u , A'B' = s' u .
We assume B' = B.

To express the transformation of any given point M of the mobile platform bearing the end effector, we will use the direct intrinsic vector method [3]. As a first step, we shall cut the platform in two parts, each one allowing an independent work of the two "parallel" arms. For the first arm we have : M -> M'1

$$\begin{bmatrix} NM'1 \\ 1 \end{bmatrix} = \begin{bmatrix} \exp(\vartheta\ w\ \wedge) & tw \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} \exp(\varphi\ w\ \wedge) & NA \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \exp(\psi\ w\ \wedge) & AB \\ 0 & 1 \end{bmatrix} \begin{bmatrix} BM \\ 1 \end{bmatrix}$$

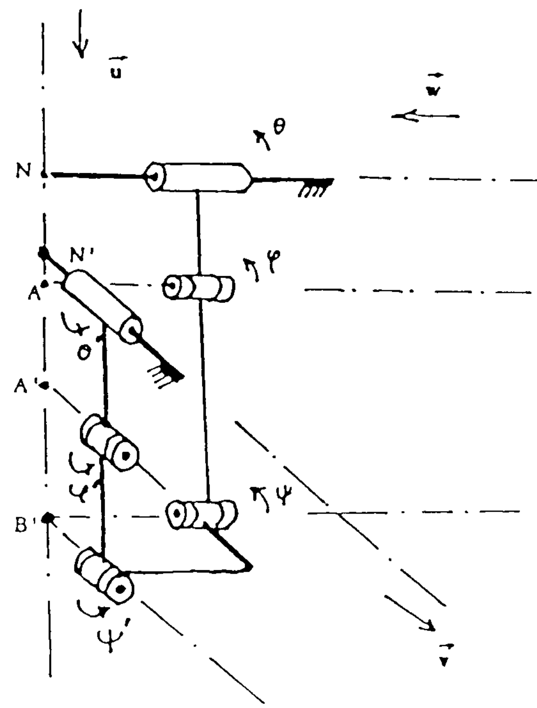

Fig. 4. Initial configuration of the robot

$$NM'1 = t\, w + \exp(\vartheta\, w\hat{})\{ NA + \exp(\varphi\, w\hat{}) [AB + \exp(\psi\, w\hat{})\, BM] \}$$

After some calculations we obtain :

$$NM'1 = [r \cos\vartheta + s \cos(\vartheta + \varphi)]\, u + [r \sin\vartheta + s \sin(\vartheta + \varphi)]\, v + t\, w + \exp[(\vartheta + \varphi + \psi) w\hat{}]\, BM$$

Analogous calculations for the second arm lead to the transformation : M -> M'2

$$N'M'2 = [r' \cos\vartheta' + s' \cos(\vartheta' + \varphi')]\, u + t'\, v + - [r' \sin\vartheta' + s' \sin(\vartheta' + \varphi')]\, w + + \exp[(\vartheta' + \varphi' + \psi') w\hat{}]\, B'M$$

As a matter of fact, there is no cut in the mobile platform :

$NM'1 = NM'2 \quad \forall$ point M,

$NM'2 = NN' + N'M'2 = p\, u + N'M'2$, =>

$$r \cos\vartheta + s \cos(\vartheta + \varphi) = p + r' \cos\vartheta' + s' \cos(\vartheta' + \varphi') \quad (1)$$

$$r \sin\vartheta + s \sin(\vartheta + \varphi) = t' \quad (2)$$

$$t = - r' \sin\vartheta' - s' \sin(\vartheta' + \varphi') \quad (3)$$

$$\exp[(\vartheta + \varphi + \psi) w\hat{}]\, BM = \exp[(\vartheta' + \varphi' + \psi') w\hat{}]\, BM \Rightarrow$$

$$\vartheta + \varphi + \psi = \vartheta' + \varphi' + \psi' \quad (4)$$

$$\vartheta + \varphi + \psi = 2k\pi \quad (5)$$

These last two equations point out the absence of rotation for the mobile platform which is translating as predicted by the general Group Theory.

We have 5 scalar equations for 8 variable parameters $t, \vartheta, \varphi, \psi, t', \vartheta', \varphi', \psi'$. So, only 3 parameters are free.

All points of the mobile platform have equal trajectories.

The coordinates of point Bm, which is the transformed of point B, are given by :

$$NBm = [r \cos\vartheta + s \cos(\vartheta + \varphi)]\, u + [r \sin\vartheta + + s \sin(\vartheta + \varphi)]\, v + t\, w .$$

We choose t, t' and ϑ as the servomotorized parameters :

$$NBm = [r \cos\vartheta + \delta \sqrt{s^2 - (t' - r \sin\vartheta)^2}\,]\, u + + t'\, v + t\, w , \qquad \delta = \pm 1$$

VII. CONCLUSIONS

The kinematic principle of displacement subgroups intersection is a powerful tool for the synthesis of new parallel structure robots.

Our interest is focused on 3 degrees of freedom "parallel" robots capable of pure spatial translation movements. These robots have, on the one hand, the advantages of classical "parallel" robots for positioning precision, rapidity and fixed motors location, which may lead to important industrial applications : high-speed assembling or distributing processes for lightweight objects; clean room work, for the motorized part of the robot can be easily isolated; low cost of the mechanical structure, and others to be verified. On the other hand, the fact of considering purposely translation movements offers other advantages related to the fact that rotation, when not strictly necessary as in many industrial applications, easily generates control and stability problems for mechanisms.

However, some difficulties that we have not taken into account, may arise when we consider these kinds of robots, as for example :

1) special configurations with mobility changes can exist (these are called singular configurations)

2) friction has to be avoided because it produces negative effects.

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